TEACHERS' RECRUITMENT BOARD
WRITTEN COMPETITIVE EXAMINATION
FOR POST GRADUATE ASSISTANTS (2006 - 2007)

TIME ALLOWED : 3 HOURS **MATHEMATICS** MAX. MARKS : 150

1. **In Indian history, who is known as the 'Indian Napoleon'?**
A) Asoka B) Chandragupta II
C) Chanakya D) Samudragupta

2. **Who of the following is associated with the theory of "Laissez-faire" in Economics?**
A) Malthus B) Marshall
C) Adam Smith D) Keynes

3. **The boundary line between India and China is known as**
A) Radcliffe line B) Durand line
C) McMahon line D) Maginot line

4. **Which of the following countries is called the "Land of White Elephants"?**
A) Malaysia B) Thailand
C) Canada D) Ethiopia

5. **Who was the founder of Brahmo Samaj?**
A) Raja Rammohan Roy
B) Rabindranatha Tagore
C) Keshab Chandra Sen
D) M.G. Ranade

6. **Malaria is caused by**
A) Plasmodium B) Virus
C) DNA D) Bacterium

7. **Article 14 of the Constitution of India deals with**
A) Equality before law
B) Abolition of untouchability
C) Freedom of speech
D) Freedom of religion

8. **Dynamo is a device for converting**
A) electricity to mechanical energy
B) mechanical energy to electrical energy
C) magnetism to electricity
D) electricity to magnetism

9. **Which of the following dynasties was *not* in power during the Sangam age?**
A) Pandyas B) Cheras
C) Cholas D) Pallavas

10. **Which country did Italy beat in the finals of the FIFA World Cup 2006?**
A) Germany B) France
C) Portugal D) Spain

11. **A useful teaching-learning method for slow learners is**
A) Lecture B) Self-learning
C) Memorising D) Group learning

12. **There is a story about a fox, who unable to reach some grapes, proclaimed that they were sour. This is a kind of**
A) intellectualization
B) rationalization
C) negativism D) egocentrism

13. **Attempts to train defectives and delinquents, so as to make them, as far as possible, useful and efficient members of the community is called**
A) Remedial instruction
B) Programmed instruction
C) Physical instruction
D) Religious instruction

14. **In an intelligence test a ten year old boy is found to have a mental age of 11. This I.Q. is calculated as**
A) 100 B) 120
C) 110 D) 90

15. **DIET stands for**
A) District Institute for Employment of Teachers
B) District Institute of Education and Training
C) District Institute of Elementary Teacher Education
D) District Institute of Educational Technology

16. **Self actualisation is defined as "the full development of personal potential" by**
A) Rotter B) Maslow
C) McCleland D) Hull

17. **Educationist Froebel is**
A) an idealist B) a naturalist
C) a realist D) a pragmatist

18. **School started by Madam Montessori was known as**
A) Children's House
B) Boy's School
C) Summer Hill School
D) Girls' School

19. **MLL represents**
A) Marginal Level of Learning
B) Maximum Level of Learning
C) Motor Learning Level
D) Minimum Level of Learning

20. **The name of the educational policy of Gandhiji is**
A) Social Education
B) Basic Education
C) Technical Education
D) Rural Education

21. **A period showing no progress in a learning curve is termed as**
A) error B) inhibition
C) plateau D) terminal point

22. **Group factor theory of intelligence was proposed by**
A) Spearman B) Thorndike
C) Thurstone D) Guilford

23. **I.Q. can be calculated using the formula**
A) $\frac{\text{Mental Age}}{\text{Chronological Age}} \times 100$
B) $\frac{\text{Chronological Age}}{\text{Mental Age}} \times 100$
C) $\frac{\text{Mental Age}}{\text{Chronological Age}}$
D) $\frac{\text{Chronological Age}}{\text{Mental Age}}$

24. **Which type of thinking is very essential for creativity?**
A) Positive thinking
B) Convergent thinking
C) Practical thinking
D) Divergent thinking

25. **Robert Gagne's theory of hierarchical learning consists of**
A) 7 types of learning
B) 2 types of learning
C) 8 types of learning
D) 10 types of learning

26. Which Article of the Constitution of India advocates free and compulsory school education?
A) Article 354 B) Article 45
C) Article 30 D) Article 31

27. The most effective way of character formation in students is to
A) advise the students frequently
B) narrate about the lives of great men and women
C) organise religious functions in the school
D) make them sing songs

28. A loud explosion is heard as you are teaching the class. What would you do?
A) Stay in the class and send the class leader to find the details
B) Walk out of the class to know details
C) Run to neighbouring class for information
D) Advise the students to get away from the class in an orderly manner

29. The agency which helps to improve the quality of school education at state level is
A) NCERT B) NCTE
C) SCERT D) DTE

30. Education leads to the modification of
A) Attitude B) Behaviour
C) Life D) Interest

31. Value Education means
A) Religious Education
B) Moral Education
C) Cost Education
D) Economics of Education

32. Punishment is
A) Reinforcement
B) Negative Reinforcement
C) Positive Reinforcement
D) Encouragement

33. Growth and development of the child are determined by two factors
A) heredity and school
B) school and home
C) home and society
D) heredity and environment

34. Learning in free atmosphere was advocated by
A) Montessori B) Gagne
C) J. Krishnamurthy D) Gandhiji

35. Thematic Apperception Test (TAT) is conducted to test the
A) intelligence of a person
B) personality of a person
C) memory of a person
D) achievement of a person

36. There is a tendency for all of us to seek our faults in others' is termed as
A) introjection B) repression
C) projection D) rationalisation

37. What is the principle behind individualised instruction?
A) Reinforcement and learning
B) Accommodation
C) Adaptation D) Schemes

38. Who advocated the method of 'Learning by doing'?
A) A.S. Neil B) John Dewey
C) Bertrand Russell D) Kilpatrick

39. Which of the following plays the major role in social development of a child?
A) School B) Family
C) Society D) Neighbours

40. An objective factor which determines attention in the classroom is
A) interest B) novelty
C) sentiment D) attitude

41. The general solution of $\left(\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2\right)y = e^{5x}$ is
A) $c_1e^{x} + c_2e^{2x} + \frac{1}{12}e^{5x}$
B) $c_1e^{-x} + c_2e^{2x} + \frac{1}{9}e^{5x}$
C) $c_1e^{x} + c_2e^{2x}$
D) $c_1e^{-x} + c_2e^{2x} + \frac{1}{12}\cos x$

42. The general solution of initial value problem $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$, $y(0) = 3$, $y'(0) = 5$ is
A) $2e^{4x} + e^{-3x}$ B) $e^{-4x} + e^{-3x}$
C) $2e^{-4x} + e^{3x}$ D) $e^{-4x} + 2e^{-3x}$

43. The Bessel equation of order 2 is
A) $y'' + xy' + (x^2 + 4)y = 0$
B) $xy'' + x^2y' + (x^2 - 2)y = 0$
C) $x^2y'' + xy' + (x^2 - 4)y = 0$
D) $y'' + x^2y' + (x^2 - 4)y = 0$

44. If $y_1(x) = \sin x$ and $y_2(x) = \cos x$ are solutions of $\frac{d^2y}{dx^2} + y = 0$ the Wronskian of y_1 and y_2 is
A) 2 B) 1
C) 0 D) –1

45. The Hermite polynomial of degree n is
A) $(-1)^{2n} e^{t} \frac{d^n}{dt^n}\left(e^{t^2}\right)$
B) $(-1)^{n} e^{t^2} \frac{d^n}{dt^n}\left(e^{-t^2}\right)$
C) $(-1)^{n} e^{2t} \frac{d^n}{dt^n}\left(e^{-t^2}\right)$
D) $(-1)^{n} e^{-t} \frac{d^n}{dt^n}\left(e^{-t^2}\right)$

46. The condition of integrability of the total differential equation $P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz = 0$ is
A) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial x}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial y}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
B) $P\left(\frac{\partial Q}{\partial x} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial y} - \frac{\partial P}{\partial x}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
C) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
D) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial y} - \frac{\partial P}{\partial x}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

47. If $(x-y)\,dx - x\,dy - z\,dz = 0$ is integrable, solution of this equation is
A) $x^2 - 2xy + z^2 = c$ B) $x^2 + 2xy - z^2 = c$
C) $x^2 + 2xy - 2z = c$ D) $2x + 2xy - z^2 = c$

48. The general solution of PDE $xzp + yzq = xy$ is
A) $\phi\left(\frac{x}{y}, -\frac{y}{z}\right) = 0$
B) $\phi\left(\frac{x}{y^2}, \frac{y}{z}\right) = 0$
C) $\phi\left(\frac{x}{y}, xy - z^2\right) = 0$
D) $\phi\left(\frac{x^2}{y^2}, -\frac{y}{z}\right) = 0$

49. PDE formed by eliminating arbitrary constants a and b from Z = ax+by+ab is
A) $z = px + qy + pq$ B) $z = px + qy + p^2$
C) $z = px - qy + pq$ D) $z = px + qy - pq$

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50. **PDE obtained by eliminating the arbitrary function from $Z = f(x^2 - y^2)$ is**

A) $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$ B) $\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$

C) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

D) $\frac{1}{2x}\frac{\partial z}{\partial x} - \frac{1}{2y}\frac{\partial z}{\partial y} = 0$

51. **For a Chi-square test, in a r×s contingency table, the number of degrees of freedom taken is**

A) rs – 1 B) (r – 1) (s – 1)
C) r + s – 1 D) rs + 1

52. **If σ_x, σ_y and σ_{x-y} are the standard deviations of x, y and $x - y$ respectively then the correlation coefficient r between x and y is given by**

A) $\frac{\sigma_x^2 + \sigma_y^2 - 2\sigma_{x-y}^2}{2\sigma_x\sigma_y}$ B) $\frac{\sigma_x^2 + \sigma_y^2 + 2\sigma_{xy}^2}{2\sigma_x\sigma_y}$

C) $\frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$ D) $\frac{\sigma_x^2 + \sigma_y^2 + \sigma_{x-y}^2}{2\sigma_x\sigma_y}$

53. **If r is the correlation coefficient of the random variables x and y, then the regression coefficient of x and y is**

A) $r\frac{\sigma_x}{\sigma_y}$ B) $r\frac{\sigma_y}{\sigma_x}$

C) $\frac{\sigma_x}{r\sigma_y}$ D) $\frac{\sigma_y}{r\sigma_x}$

54. **If ρ is the correlation coefficient of independent variables x and y then**

A) $\rho = -1$ B) $\rho = 0.5$
C) $\rho = 0$ D) $\rho = 1$

55. **For a t-test, the 95% confidence limits of the population mean (μ) with sample size n and sample S.D. s is given by**

A) $\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n}}$ with n degrees of freedom

B) $\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n-1}}$ with n degrees of freedom

C) $\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n-1}}$ with n – 1 degrees of freedom

D) $\bar{x} \pm t_{0.05}\frac{s}{\sqrt{n}}$ with n – 1 degrees of freedom

56. **If x_i (i = 1, 2, ...n) is a random sample of size n from a population then its sample variance S^2 is given by**

A) $S^2 = \sum_{i=1}^{n} \frac{x_i - \bar{x}}{n}$

B) $S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$

C) $S^2 = \sum_{i=1}^{n} \frac{(x_i^2 - \bar{x})}{n}$

D) $S^2 = \sum_{i=1}^{n} \frac{x_i^2 - \bar{x}}{n-1}$

57. **If χ_1^2 and χ_2^2 are two independent chi-square variates with degrees of freedom γ_1 and γ_2 respectively, then F-Statistic is defined by**

A) $\frac{\chi_1^2}{\chi_2^2}\frac{\gamma_2}{\gamma_1}$ B) $\frac{\chi_1^2}{\chi_2^2}\frac{\gamma_1}{\gamma_2}$

C) $\frac{\chi_1^2}{\chi_2^2}\gamma_1\gamma_2$ D) $\frac{1}{\gamma_1\gamma_2}\frac{\chi_1^2}{\chi_2^2}$

58. **The equation of two regression lines are $5x - y = 22$ and $64x - 45y = 24$. The mean values of x and y are given by**

A) 3 and 4 B) 4 and 3
C) 6 and 8 D) 8 and 6

59. **The standard error of mean of a random sample of size n from a population with variance σ^2 is**

A) $\frac{\sigma^2}{n}$ B) $\frac{\sigma}{\sqrt{n}}$

C) $\frac{\sqrt{\sigma}}{n}$ D) $\frac{\sigma}{n}$

60. **A particular integral of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \sin 2x$ is**

A) $\frac{1}{8}\sin 2x$ B) $\frac{1}{8}\cos 2x$

C) $-\frac{1}{4}\sin 2x$ D) $\frac{1}{4}\cos 2x$

61. **If $A_1, A_2, ...\ A_n$ are n mutually exclusive and exhaustive events such that $P(A_i)>0$ for i = 1, 2, ... n, let B be any event with $P(B)>0$, then $P(A_i / B)$ is equal to**

A) $\frac{P(A_i)P(B/A_i)}{\sum_{j=1}^{n} P(A_j)P(B/A_j)}$

B) $\frac{P(B)P(B/A_i)}{\sum_{j=1}^{n} P(A_j)P(B/A_j)}$

C) $\frac{P(A_i)}{\sum_{j=1}^{n} P(A_j)P\left(\frac{B}{A_j}\right)}$

D) $\frac{P(B)}{\sum_{j=1}^{n} P(A_j)P(B/A_j)}$

62. **If $f(x) = \frac{A}{\pi}\frac{1}{16 + x^2}$, $-\infty < x < \infty$ be a pdf of a continuous random variable x, then the value of A is**

A) 16 B) 8
C) 4 D) 1

63. **The distribution function F(x) of a random variable is**

A) a decreasing function
B) a non-decreasing function
C) a constant function
D) increasing first and then decreasing

64. **The mean of a Binomial distribution is 5 and its standard deviation is 2. Then the values of n and p are**

A) $\left(\frac{4}{5}, 25\right)$ B) $\left(25, \frac{4}{5}\right)$

C) $\left(\frac{1}{5}, 25\right)$ D) $\left(25, \frac{1}{5}\right)$

65. **If X is a continuous random variable, then Var (4X+3) is**

A) 4 Var (X) B) 16 Var (X)
C) 16 Var (X)+9 D) 16 Var (X)+3

66. **If X is a Poisson random variable such that $E(X^2) = 30$, then the variance of X is**

A) 6 B) 5
C) 30 D) 25

67. **If X is a random variable such that its mean = median = mode, then X follows**

A) Binomial distribution
B) Poisson distribution
C) Cauchy distribution
D) Normal distribution

68. **The density function of a random variable X is given by**

$$f(x) = \begin{cases} x/2 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

then $E(3X^2 - 2X)$ is

A) $\frac{10}{3}$ B) $\frac{4}{3}$

C) $\frac{3}{10}$ D) $\frac{3}{4}$

69. If χ is a Chi-square distribution with n degrees of freedom, its mean and variance are given by

A) n and 2n B) n and n
C) 2n and 2n D) 2n and n

70. If X is Cauchy random variable, then E(X) is

A) 1 B) 2
C) 0 D) does not exist

71. If H is a subgroup of a finite group G, then

A) O(H) is a divisor of O(G)
B) O(G) is a divisor of O(H)
C) O(H) is not a divisor of O(G)
D) O(G) is not a divisor of O(H)

72. If H is a subgroup of G and N is a normal subgroup of G, then

A) $H \cap N$ is a subgroup of G
B) $H \cup N$ is a subgroup of G
C) $H \cap N$ is a normal subgroup of G
D) $H \cup N$ is a normal subgroup of G

73. If G is a group of order $11^2 . 13^2$ then

A) G has one 11-sylow subgroup
B) G has one 13-sylow subgroup
C) G has one 11-sylow subgroup and one 13-sylow subgroup
D) G has one 11-sylow subgroup and one 13-sylow subgroup which are normal in G

74. A ring is a Boolean ring if

A) $x^3 = x$ B) $x^2 = x$
C) $x^3 = 1$ D) $x^2 = e$

75. If R is a ring and $a \subseteq R$ and r(a) = $\{x \in R \mid ax = 0\}$, then

A) r(a) is a right ideal of R
B) r(a) is a left ideal of R
C) r(a) is a two-sided ideal of R
D) r(a) is not a two-sided ideal of R

76. The ideal A = (a_0) is maximal ideal of the ring R. Then

A) A = R
B) A $\neq$ R
C) a_0 is a prime element of R
D) a_0 is not a prime element of R

77. The linear transformation T is unitary if

A) (vT, vT) = (v, v) d $v \in V$
B) (vT, vT) = (v, v) for at least one $v \in V$
C) (vT, v) = (v, v)
D) (vT, uT) = (v, u)

78. If u, $v \in V$, then u is orthogonal to v, if

A) (u, v) = 0 B) (u, v) = 1
C) $||u|| = 1$ D) $||u|| = ||v||$

79. The characteristic roots of a Hermitian matrix are

A) all complex B) all real
C) all rational D) all irrational

80. If V is a finite dimensional vector space over a field F, then

A) V has a unique basis
B) any linearly independent subset of V can be extended to a basis
C) any subset of V can be extended to a basis
D) any linearly independent set can be extended to form an orthonormal basis

81. The transformation f(z) = $\bar{z}$ is

A) isogonal
B) conformal
C) neither isogonal nor conformal
D) bilinear

82. If a function f(z) is analytic inside and on a closed curve C, then Cauchy's integral formula gives the value of the function

A) at an exterior point
B) at an interior point
C) at a boundary point
D) at all points

83. The value of the line integral $\int \bar{z}\, dz$ from z = 0 to z = 4 + 2i along the curve z = t^2 + it is

A) $\frac{10-8i}{3}$ B) $\frac{5-4i}{3}$
C) $5 - \frac{4i}{3}$ D) $10 - \frac{8i}{3}$

84. The Taylor's series expansion of $\tan^{-1}$ z about z = 0 is

A) $1-\frac{z^2}{2}+\frac{z^4}{4}-........$
B) $z-\frac{z^3}{3!}+\frac{z^5}{5!}-........$
C) $1-\frac{z^2}{2!}+\frac{z^4}{4!}-........$
D) $z-\frac{z^3}{3}+\frac{z^5}{5}-........$

85. The point z = –1 for the function $(z-2)\sin\frac{1}{z+1}$ is

A) a removable singularity
B) a pole
C) an essential singularity
D) a zero

86. The Laurent's series expansion of $\frac{1}{(z+1)(z+3)}$ valid for the region $1<|z+1|<2$ is

A) $\frac{1}{z+1}-\frac{1}{2}+\frac{z+1}{4}-\frac{(z+1)^2}{16}+......$
B) $\frac{1}{2(z+1)}+\frac{1}{4}-\frac{z+1}{8}+\frac{(z+1)^2}{16}+.....$
C) $\frac{1}{2(z+1)}-\frac{1}{4}+\frac{z+1}{8}-\frac{(z+1)^2}{16}+.....$
D) $-\frac{1}{z+1}+\frac{1}{2}-\frac{z+1}{4}+\frac{(z+1)^2}{16}-.....$

87. The residue of $\frac{1}{(z^2+1)^3}$ at z = i is

A) $-\frac{3i}{16}$ B) $\frac{i}{16}$
C) $\frac{3i}{16}$ D) $-\frac{i}{16}$

88. If A and B are two independent events such that P(A) = 0.5 and P($A \cup B$) = 0.8, then P(B) =

A) 0.3 B) 0.4
C) 0.5 D) 0.6

89. Two cards are drawn from a pack of 52 cards in succession. Then the probability that both are queens when the first card is not replaced, is

A) $\frac{1}{169}$ B) $\frac{8}{52}$
C) $\frac{1}{221}$ D) $\frac{2}{221}$

90. If A and B are any two events and P(A) = 0.35, P(B) = 0.73, P($A \cap B$) = 0.14, then $P\left(A\overline{\cup}B\right)$ is

A) 0.94 B) 0.59
C) 0.86 D) 0.06

91. If x and y are any two vectors in the Hilbert space H, then

A) $(x, y) \leq \|x\| \, \|y\|$
B) $|(x, y)| \leq \|x\| \, \|y\|$
C) $|(x, y)| \geq \|x\| \, \|y\|$
D) $(x, y) \geq \|x\| \, \|y\|$

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92. Let S be a non-empty subset of a Hilbert space H. The set of all vectors orthogonal to S is

A) H itself

B) a linear subspace of H

C) a closed linear subspace of H

D) an empty set

93. Which of the following is an orthonormal subset of l_2^n?

A) {(1, 1, ... 1), (1, 0, 1, ... 1), (1, 0, 0, 1, ... 1) ... (1, 0, 0, ... 0, 1)}

B) {(1, 0, 1, ... 1), (1, 1, 0, 1, ... 1) ... (1, 1, ... 1, 0, 1)}

C) {(1, 0, ... 0), (0, 1, 1, ... 1), (1, 1, 0, 0, ... 1) ... (1, 1, ... 1)}

D) {(1, 0, 0, ... 0), (0, 1, 0, ... 0) ... (0, 0, ... 0, 1)}

94. If $\{e_t\}$ is an orthonormal set in a Hilbert space H and x is any vector in H, then the set $S = \{e_i : (x, e_i) \neq 0\}$ is

A) either empty or countable

B) S is uncountable

C) S is empty

D) S is countable

95. Let A be a Banach algebra. Let G and S be the set of all regular elements and the set of all singular elements of A respectively. Then

A) G is open and S is closed

B) G is closed and S is open

C) G is closed set

D) S is open set

96. Let $\sigma(x)$ be the spectrum of x where x is an element of Banach algebra A. Then

A) $\sigma(x) = \{\lambda : x - \lambda I \text{ is regular}\}$

B) $\sigma(x)$ is always non-empty

C) $\sigma(x)$ is empty

D) $\sigma(x^n) = \sigma(x)^{n-1}$

97. If $\lim\limits_{n\to\infty} Z_n = k$, then $\lim\limits_{n\to\infty} \frac{1}{n}$ $(Z_1 + \ldots + Z_n)$ is equal to

A) 0 B) ∞

C) 1 D) k

98. The region of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!\, Z^n}{n^n}$ is

A) $|Z| < e^2$ B) $|Z| < e$

C) $|Z| < \frac{1}{e}$ D) $|Z| < \frac{1}{e^2}$

99. The cross ratio (z_1, z_2, z_3, z_4) is the image of z_1 under the linear transformation which carries z_2, z_3, z_4 onto

A) 0, 1, ∞ B) 1, 0, ∞

C) ∞, 1, 0 D) ∞, 0, 1

100. The bilinear transformation which maps z = 0, –i, –1 onto w = i, 1, 0 respectively is

A) $i\left(\frac{z+1}{z-1}\right)$ B) $i\left(\frac{z-1}{z+1}\right)$

C) $-i\left(\frac{z+1}{z-1}\right)$ D) $-i\left(\frac{z-1}{z+1}\right)$

101. The number of non-basic variables in the balanced transportation problem with 4 rows and 5 columns is

A) 12 B) 20

C) 11 D) 8

102. For a constant hazard model, the mean time to failure is given by

A) λ B) $\frac{1}{\lambda}$

C) $\lambda^2 + \lambda$ D) $\frac{1}{\lambda^2}$

103. A system has 100 units in series, each having a reliability of 0.98, the reliability of the system is

A) $(0.98)^{100}$ B) $[1 - (1-0.98)^{100}]$

C) $[1 - 0.98]^{100}$ D) $[100]^{0.98}$

104. If the total float of an activity 3 – 4 is 18 and the latest and earliest occurrences of the events 3 and 4 are 15, 12 and 22, 10 respectively, the free float 3–4 is

A) 10 B) 15

C) 12 D) 6

105. If F = {f(i, j)} is the flow function and C = {c(i, j)} is the capacity function then which of the following is true?

A) $f(i, j) \leq c(i, j)$ B) $c(i, j) \leq f(i, j)$

C) $f(i, j) + c(i, j) = 0$

D) $f(i, j) \not< c(i, j)$

106. Let M be a subspace of a normed linear space N. The set of all cosets $\{x+M : x \in N\}$ is a normed space in the quotient form if

A) M is an open subspace of N

B) M = N

C) M is a closed subspace of N

D) M is a finite subspace of N

107. Let $T : N_1 \to N_2$ be a continuous linear transformation of a normed linear space N_1 into another normed linear space N_2. Then

A) $x_n \to 0 \Rightarrow T(x_n) \not\to 0$

B) $\|T(x)\| \leq \|x\| \ \forall x \in N$

C) there exists a real number $k \geq 0$ with $\|T(x)\| \leq k\|x\| \ \forall x \in N$

D) there exists a real number $k \geq 0$ with the property that $\|T(x)\| \geq k\|x\| \ \forall x \in N$

108. Let N be a normed space and x_0 is a non-zero vector in N, then there exists a functional f_0 in the conjugate space of N such that

A) $f_0(x_0) = x_0$ and $\|f_0\| = 1$

B) $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$

C) $f_0(x_0) = 1$ and $\|f_0\| \neq 1$

D) $f_0(x_0) = 1$ and $\|f_0\| \geq 1$

109. Let P be a projection on a Banach space B. Let M = {P(x) : $x \in$ B). Then M is

A) the null space of the operator I – P

B) the range space of the operator I – P

C) the null space of M

D) singleton

110. Let X and Y be Banach spaces, the graph of a linear transformation T of X into Y is closed if and only if

A) T is onto

B) T is one to one and onto

C) T is continuous

D) T is one to one

111. The Dupin's Indicatrix is

A) a straight line

B) a circle

C) a conic section

D) a pair of straight lines

112. The asymptotic lines are given by the condition

A) $d\bar{r} \times d\bar{N} = 0$ B) $d\bar{r} . d\bar{N} = a$

C) $d\bar{r} \times d\bar{N} = \bar{a}$ D) $d\bar{r} . d\bar{N} = 0$

113. A developable surface is

A) one parameter family of planes

B) one parameter family of curves

C) one parameter family of spheres

D) one parameter family of straight lines

114. The canonical equations of geodesics are

A) $\frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right) - \frac{\partial T}{\partial u} = 0$

$\frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right) - \frac{\partial T}{\partial v} = 0$

B) $\frac{d}{ds}\left(\frac{\partial T}{\partial u}\right) - \frac{\partial T}{\partial v} = 0$

$\frac{d}{ds}\left(\frac{\partial T}{\partial v}\right) - \frac{\partial T}{\partial u} = 0$

C) $\frac{d}{ds}\left(\frac{\partial T}{\partial v}\right) - \frac{\partial T}{\partial v'} = 0$

$\frac{d}{ds}\left(\frac{\partial T}{\partial u}\right) - \frac{\partial T}{\partial u'} = 0$

D) $\frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right) - \frac{\partial T}{\partial v} = 0$

$\frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right) - \frac{\partial T}{\partial u} = 0$

115. A linear constraint $a_1x_1+a_2x_2 \geq b$ is equivalent to

A) $a_1 x_1 - a_2 x_2 \leq b$

B) $-a_1 x_1 - a_2 x_2 \leq -b$

C) $-a_1 x_1 + a_2 x_2 \geq b$

D) $-a_1 x_1 + a_2 x_2 \geq -b$

116. One of the characteristics of the standard form of linear programming problem is

A) all variables are negative

B) the right hand side element of each contraint equation is negative

C) the objective function is of the maximization or minimization type

D) the right hand side element of each constraint equation is positive

117. If the Primal Problem is
Min Z = CX
subject to the constraints
AX = b and X≥0, then the corresponding dual problem is

A) Min $Z^* = b^T W$
subject to $A^T W \geq C^T$ and $W \geq 0$

B) Min $Z^* = b^T W$
subject to $A^T W \geq C^T$ and W is unrestricted

C) Max $Z^* = b^T W$
subject to $A^T W \leq C^T$ and W is unrestricted

D) Min $Z^* = b^T W$
subject to $A^T W = C^T$ and W is unrestricted

118. If the minimax value = maximum value then the corresponding pure strategies are called 'optimal' strategies and the game is said to have a

A) critical point B) constant point

C) maximal point D) saddle point

119. For the game with the following pay-off matrix, $\begin{array}{cc} & B \\ A & \begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix} \end{array}$ **the value of the game is**

A) 6 B) $-\frac{3}{4}$

C) 0 D) –3

120. At a public telephone booth, arrivals are considered to be Poisson with an average inter-arrival time of 12 min. The length of phone call is 4 min. The average length of queues formed from time to time is

A) 4 B) 6

C) 10 D) 1.5

121. The theta function θ given by $\theta(x) = \sum_{n=-\infty}^{\infty} e^{-\pi n^2 x}$, $x>0$ can also be given as the transformation equation

A) $\theta(x) = \frac{1}{\sqrt{x}}\theta\left(\frac{1}{\sqrt{x}}\right),\ x>0$

B) $\theta(x) = \frac{1}{\sqrt{x}}\theta\left(\frac{1}{x}\right),\ x>0$

C) $\theta(x) = \frac{1}{x}\theta(x),\ x>0$

D) $\theta(x) = \frac{1}{x}\theta\left(\frac{1}{x}\right),\ x>0$

122. The Fourier transform of the function F(x) defined by
$F(x) = \begin{cases} 1, & |x|<a \\ 0, & |x|>a \end{cases}$ **is**

A) $\sqrt{\frac{1}{\pi}}\ \frac{\cos pa}{p}$ B) $\frac{1}{\sqrt{2\pi}}\ \frac{\cos pa}{p}$

C) $\sqrt{\frac{1}{\pi}}\ \frac{\sin pa}{p}$ D) $\sqrt{\frac{2}{\pi}}\ \frac{\sin pa}{p}$

123. If f(x) and g(x) have Fourier coefficients a_n, b_n and A_n, B_n respectively in [$-\pi_0$, π] then
$\sum_{n=1}^{\infty}(a_nA_n + b_nB_n) + \frac{1}{2}a_0A_0 =$

A) $\frac{1}{\pi}\int_0^{\pi} f(x)g(x)\,dx$

B) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\,dx$

C) $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\,dx$

D) $\frac{2}{\pi}\int_0^{\pi} f(x)g(x)\,dx$

124. If $\bar{R}$ is the position vector of any point in the osculating plane and if the curve is given in terms of an arbitrary parameter u, then the equation to the osculating plane is

A) $\left[\bar{R},\ \dot{\bar{r}},\ \ddot{\bar{r}}\right] = 0$

B) $\left[\bar{R} - \bar{r},\ \ddot{\bar{r}},\ \dddot{\bar{r}}\right] = 0$

C) $\left[\bar{R} - \bar{r},\ \dot{\bar{r}},\ \ddot{\bar{r}}\right] = 0$

D) $\left[\bar{R} - \dot{\bar{r}},\ \ddot{\bar{r}},\ \dddot{\bar{r}}\right] = 0$

125.The matrix of the coefficient of Serret-Frenet formula is

A) $\begin{bmatrix} 0 & k & 0 \\ -k & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & \tau & 0 \\ -\tau & 0 & k \\ 0 & -k & 0 \end{bmatrix}$

C) $\begin{bmatrix} 0 & -\tau & 0 \\ \tau & 0 & -k \\ 0 & k & 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 & -k & 0 \\ k & 0 & -\tau \\ 0 & \tau & 0 \end{bmatrix}$

126.The radius of spherical curvature R is given in terms of the radius of curvature ρ of the curve at P and σ the reciprocal of the torsion is given by

A) $R^2 = \sigma^2 + (\rho'\rho)^2$

B) $R^2 = \rho^2 + (\rho'\sigma)^2$

C) $R^2 = \rho'^2 + (\rho\sigma)^2$

D) $R^2 = \sigma^2 + (\rho'/\sigma)^2$

127.The necessary and sufficient condition for a curve to be a helix is that

A) the curvature is constant

B) the torsion is constant

C) the product of the curvature and the torsion is constant

D) the ratio of the curvature and the torsion is constant

128.The first fundamental form is

A) $H^2 = EG - F^2$

B) $ds^2 = E^2 du^2 + 2F^2 du\, dv + G^2 dv^2$

C) $ds^2 = E du^2 + 2F du\, dv + G dv^2$

D) $H = EG - F^2$

129.The element of surface ds is given by

A) $ds = H^2 \sqrt{du^2 + dv^2}$

B) $ds = H^2 du\, dv$

C) $ds = H \sqrt{du^2 + dv^2}$

D) $ds = H du\, dv$

130.The necessary and sufficient condition for a curve to be a geodesic on a surface is

A) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

B) $U\frac{\partial T}{\partial \dot{u}} - V\frac{\partial T}{\partial \dot{v}} = 0$

C) $U\frac{\partial T}{\partial v} - V\frac{\partial T}{\partial u} = 0$

D) $U\frac{\partial T}{\partial u} - V\frac{\partial T}{\partial v} = 0$

131.The set of all irrational numbers is

A) of first category

B) of second category

C) neither first category nor second category

D) both of first category and second category

132.The set of rational numbers is of measure

A) ∞ B) 1

C) 0 D) 2

133.An orthonormal system of complex valued functions on every interval [k, k + 2π] where k∈ℝ is

A) $\frac{e^{inx}}{\sqrt{2\pi}}$, $n = 0, 1, 2, ...$

B) $\frac{e^{inx}}{\sqrt{\pi}}$, $n = 0, 1, 2, ...$

C) $\frac{e^{i2nx}}{\sqrt{2\pi}}$, $n = 0, 1, 2, ...$

D) $\frac{e^{i2nx}}{\sqrt{2\pi}}$, $n = 1, 3, 5, ...$

134.Let I = [0, 2π]. Let f∈L ([0, 2π]). The Fourier series generated by *f* is given by $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$. Then

A) $a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\, dt,\ b_n = \frac{1}{\pi}\int_0^{2\pi} f^n(t)\sin nt\, dt$

B) $a_n = \frac{1}{2\pi}\int_0^{2\pi} f(t)\cos nt\, dt,\ b_n = \frac{1}{2\pi}\int_0^{2\pi} f(t)\sin nt\, dt$

C) $a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt\, dt,\ b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\, dt$

D) $a_n = \frac{1}{2\pi}\int_0^{2\pi} f(t)\cos nt\, dt,\ b_n = \frac{1}{2\pi}\int_0^{2\pi} f^n(t)\sin nt\, dt$

135.Let $\{\phi_n\}_{n=0}^{\infty}$ be an orthonormal set on an interval I. Let f∈L^2 (I) and f(x) ~ $\sum_{n=0}^{\infty} C_n\phi_n(x)$. Then

A) $\sum_{n=0}^{\infty} |C_n|^2 \le \|f\|$

B) $\sum_{n=0}^{\infty} |C_n| \le \|f\|^2$

C) $\sum_{n=0}^{\infty} |C_n|^2 \le \|f\|^2$

D) $\sum_{n=0}^{\infty} |C_n|^2 = \|f\|^2$

136.Let *f*∈L ([0, 2π]) and let *f* be periodic with period 2π. Let s(*x*) = $\lim_{t\to 0+} \frac{f(x+t)+f(x-t)}{2}$ whenever the limit exists. Then

A) for each *x*, for which s(*x*) is defined, the Fourier series generated by f has (C, 1) sum s(*x*)

B) for each *x*, for which s(*x*) is defined, the Fourier series generated by f has sum s(*x*)

C) the function f will not generate a Fourier series

D) f generates a Fourier series with (C, 1) sum $s^2(x)$

137.Let *f* and g be Lebesgue integrable functions on (–∞, ∞). Then the convolution of *f* and g is

A) $\int_{-\infty}^{+\infty} f(t)g(t)\,dt$

B) $\int_{-\infty}^{+\infty} f(t)g(x-t)\,dt$

C) $\int_0^{\infty} f(t)g(t-x)\,dt$

D) $\int_{-\infty}^{+\infty} f(x+t)g(t)\,dt$

138.Let f(t) = $\frac{1}{\sqrt{t}}$ and g(t) = $\frac{1}{\sqrt{1-t}}$ 0<t<1 and let f(t) = g(t) = 0 if t≤0 or if t≥1, then

A) f is continuous at t = 0

B) the Lebesgue integral $\int_{-\infty}^{+\infty} f(t)\,dt$ does not exist

C) the Lebesgue integral $\int_{-\infty}^{+\infty} g(t)\,dt$ does not exist

D) the convolution integral of f and g does not exist

139. Let $\mathbb{R} = (-\infty, \infty)$. Assume that $f \in L^2(\mathbb{R})$ and $g \in L^2(\mathbb{R})$. Then the convolution $h(x)$ of f and g

A) exists for each $x \in \mathbb{R}$ and h is bounded on $\mathbb{R}$
B) exists for each $x \in \mathbb{R}$ and h is unbounded
C) does not exist
D) is unbounded

140. Let f be a non-negative function such that the integral $\int_{-\infty}^{+\infty} f(x)\,dx$ exists as an improper Riemann Integral. Assume that f increases on $(-\infty, 0]$ and decrease on $[0, \infty)$. Then $\sum_{m=-\infty}^{\infty} \frac{f(m+) + f(m-)}{2} =$

A) $\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\, e^{-2\pi int}\, dt$
B) $\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\, e^{-2\pi nt}\, dt$
C) $\sum_{n=0}^{\infty} \int_{0}^{\infty} f(t)\, e^{-2\pi int}\, dt$
D) $\sum_{-\infty}^{0} f(t)\, e^{-2\pi int}\, dt$

141. If the field F has p^m elements where p is a prime and m is a positive integer, which one of the following is not true?

A) Every $a \in F$ satisfies $a^{p^m} = a$
B) $x^{p^m} - x$ in $F[x]$ factors in $F[x]$ as $x^{p^m} - x = \underset{\lambda \in F}{\pi}(x - \lambda)$
C) There is a proper subfield K of F such that $x^{p^m} - x$ splits in K
D) F is the splitting field of $x^{p^m} - x$

142. Which of the following is true?

A) Every integer $n > 1$ is either a prime or a product of primes
B) For every real x there is a positive integer n such that $n < x$
C) The set S of intervals with rational end-points is an uncountable set
D) The closed interval [0, 1] is a countable set

143. Which of the following is true?

A) Every bounded sequence is convergent
B) A sequence converges to more than one limit
C) Every convergent sequence is bounded
D) A bounded sequence may have unbounded range

144. If a bounded set S in R^n contains infinitely many points then

A) there is at least one point in R^n which is an accumulation point of S
B) there is a unique point in R^n which is an accumulation point of S
C) there is no accumulation point of S
D) S is a closed subset of R^n

145. Let $f : S \to T$ be a function from a metric space S to another metric space T. Assume that f is continuous on a compact subset of S. Then

A) f is constant
B) f has a unique fixed point
C) f is uniformly continuous
D) f is 1 – 1 and onto

146. Let $f : S \to T$ be a function from one metric space S to another metric space T. Then f is continuous on S if and only if

A) for every open set G in T, $f^{-1}(G)$ is closed in S
B) for every closed set F in T, $f^{-1}(F)$ is open in S
C) for every open set G in S, f(G) is open in T
D) for every closed set F in T, $f^{-1}(F)$ is closed in S

147. Let G be an open covering of a closed and bounded set A in R^n, then

A) an infinite subcollection of G also covers A
B) there is no finite subcollection of G covering A
C) there is a finite subcollection of G covering A
D) A has only a finite number of limit points

148. If a function f on [a, b] is continuous on [a, b], differentiable on (a, b) and $f(a) = f(b)$, then

A) there exists a unique C between a and b such that $f'(C) = 0$
B) there exists a unique C between a and b such that $f'(C) \neq 0$
C) there exists at least one C between a and b such that $f'(C) = 0$
D) there exists a unique C between a and b such that $f''(C) \neq 0$

149. If $[x]$ is the greatest integer $\leq x$, the value of $\int_0^4 [x]\,dx$ is

A) 4 B) 6
C) 8 D) 10

150. Let f be a bounded real valued function defined on [a, b]. If P* is a refinement of partition P, then L (P*, f) is

A) ≤ L (P, f) B) ≥ U (P, f)
C) ≥ L (P, f) D) ≥ U (P*, f)

POST GRADUATE ASSISTANTS (2006 - 2007) – MATHEMATICS – ANSWERS

1 .... D	2 C	3 C	4 .... B	5 .... A	6 .... A	7 .... A	8 .... B	9 .... D	10 .... B
11 .... D	12 .... A	13 .... A	14 C	15 .... B	16 .... B	17 .... B	18 .... A	19 .... D	20 .... B
21 C	22 C	23 .... A	24 .... D	25 C	26 .... B	27 .... B	28 .... B	29 .... A	30 .... B
31 .... B	32 .... B	33 .... D	34 C	35 .... B	36 C	37 .... A	38 .... B	39 C	40 .... A
41 .... A	42 .... A	43 C	44 .... D	45 .... B	46 C	47 .... *	48 C	49 .... A	50 C
51 .... B	52 C	53 .... A	54 C	55 C	56 .... B	57 .... A	58 C	59 .... B	60 .... B
61 .... A	62 C	63 .... B	64 .... D	65 .... B	66 .... B	67 .... D	68 .... A	69 .... A	70 .... D
71 .... A	72 C	73 .... D	74 .... B	75 .... A	76 C	77 .... A	78 .... A	79 .... B	80 .... B
81 .... A	82 .... B	83 .... D	84 .... D	85 C	86 C	87 .... A	88 .... D	89 C	90 .... D
91 .... B	92 C	93 .... D	94 .... D	95 .... A	96 .... B	97 .... B	98 .... B	99 .... A	100 C
101 .... A	102 .... B	103 .... A	104 .... D	105 .... *	106 C	107 C	108 .... B	109 .... A	110 C
111 C	112 .... D	113 .... A	114 .... D	115 .... B	116 C	117 C	118 .... D	119 .... B	120 .... D
121 .... B	122 .... D	123 .... B	124 C	125 .... A	126 .... B	127 .... D	128 C	129 .... D	130 .... A
131 .... B	132 C	133 .... A	134 C	135 .... D	136 .... A	137 .... B	138 .... D	139 .... A	140 .... A
141 .... D	142 .... A	143 C	144 .... A	145 C	146 .... D	147 C	148 C	149 .... B	150 C

D/Book/TRB-06-07/PG/Maths-S.N.

DETAILED ANSWERS

41. (D)

$$\left(\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2\right)y = e^{5x}$$

Auxilary equation is

$m^2-3m+2 = 0$

$(m-2)(m-1) = 0$

$\therefore m = 1, 2$

C.F. $= C_1e^x+C_2e^{2x}$

P.I. $= \dfrac{e^{5x}}{D^2-3D+2}$

$= \dfrac{e^{5x}}{5^2-3+5+2}$

$= \dfrac{e^{5x}}{25-15+2} = \dfrac{e^{5x}}{12}$

Solution

$y = CF+PI$

$\Rightarrow y = C_1e^x+C_2e^{2x}+\dfrac{e^{5x}}{12}$

42. (A)

$\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-12y=0$, $y(0) = 3$

$y'(0) = 5$

Auxillary equationi is

$m^2-m-12 = 0$

$(m-4)(m+3) = 0$

$m = 4, -3$

$\therefore$ General solutiion is

$y = Ae^{4x}+Be^{-3x}$ ... (1)

But

$y(0) = 3$

$\Rightarrow 3 = Ae^{4(0)}+Be^{-3(0)}$

$\Rightarrow 3 = A+B$ ... (2)

From (1)

$y' = 4Ae^{4x}-eBe^{-3x}$

$y'(0) = 5$

$5 = 4Ae^{4(0)}-3Be^{-3(0)}$

$\Rightarrow 5 = 4A-3B$ ... (3)

Solving (2) and (3)

$A = 2$; $B = 1$

$\therefore (1) \Rightarrow y = 2e^{4x}+e^{-3x}$

43. (C)

Bessel equation of order 2 is

$x^2y''+xy'+(x^2-4)y = 0$

44. (D)

$y_1(x) = \sin x$; $y_2(x) = \cos x$

$y_1' = \cos x$; $y_2' = -\sin x$

Wronskian of y_1 and y_2 is

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$$

$= -\sin^2x-\cos^2x$

$= -(\sin^2x+\cos^2x)$

$= -1$

45. (B)

Hermite polynomial of degree n is $(-1)^n e^{t^2}\dfrac{d^n}{dt^n}\left(e^{-t^2}\right)$

47. (*)

Consider

$(x-y)dx-yxdy = 0$

$\Rightarrow xdx-ydx-xdy = 0$

$\Rightarrow xdx-(xdy+ydx) = 0$

$\Rightarrow xdx-d(xy) = 0$

On integration

$\dfrac{x^2}{2}-xy+c = 0$

$\Rightarrow x^2-2xy+c_1 = 0$

since z is constant

$c_1 = \phi_1(z)$

$\therefore x^2-2xy+\phi(z) = c$ ... (1)

Differentiating

$2xdx-2ydx-2xdy+\phi(z)dz = 0$

$\Rightarrow (x-y)dx-xdy+\dfrac{\phi'(z)}{2}dz=0$

Comparing this with given problem

$(x-y)dx-xdy-zdz = 0$

$\dfrac{\phi'(z)}{2} = -z$

$\therefore \phi(z) = -2\left(\dfrac{z^2}{2}\right)$

$= -z^2$

$\therefore (1) \Rightarrow$

$x^2-2xy-z^2 = c$

48. (C)

$xzp+yzq = xy$

This is Lagrange's equation.

Therefore auxiliary equation is

$\dfrac{dx}{xz}=\dfrac{dy}{yz}=\dfrac{dz}{xy}$ ... (1)

$\Rightarrow \dfrac{dx}{xz}=\dfrac{dy}{yz}$

$\Rightarrow \dfrac{dx}{x}=\dfrac{dy}{y}$

integrating

$\log x = \log y+\log a$

$\log\dfrac{x}{y} = \log a$

$\therefore \dfrac{x}{y} = a$

Also (1) $\Rightarrow$

$\dfrac{ydx+xdy-2zdz}{xyz+xyz-2xyz}$

$= \dfrac{ydx+xdy-2zdz}{0}$

$= d(xy)-2zdz = 0$

$\therefore xy-z^2 = b$

Therefore the general solution is

$\phi\left(\dfrac{x}{y}, xy-z^2\right) = 0$

49. (A)

$z = ax+by+ab$ ... (1)

$p = \dfrac{\partial z}{\partial x} = a$

$q = \dfrac{\partial z}{\partial y} = b$

$\therefore (1) \Rightarrow z = px+qy+pq$

50. (C)

$z = f(x^2-y^2)$

$\dfrac{\partial z}{\partial x} = f'(x^2-y^2)\times 2x$

$\Rightarrow f'(x^2-y^2) = \dfrac{\left(\frac{\partial z}{\partial x}\right)}{2x}$ ... (1)

$\dfrac{\partial z}{\partial y} = f'(x^2-y^2)(-2y)$

$$f'(x^2-y^2) = \frac{\left(\frac{\partial z}{\partial y}\right)}{-2y} \quad \dots (2)$$

From (1) and (2)

$$\frac{\left(\frac{\partial z}{\partial x}\right)}{2x} = \frac{\left(\frac{\partial z}{\partial y}\right)}{-2y}$$

$$\Rightarrow y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$$

58. (C)

Mean of x and y are given by intersection of $5x-y=22$ and $64x-45y=24$ on solving

$$64x-45y = 24$$
$$225x-45y = 990$$
$$\Rightarrow x = 6$$
$$\therefore y = 8$$

60. (B)

Particular integral $= \frac{\sin 2x}{D^2-4D+4}$

$$= \frac{\sin 2x}{-4-4D+4} = \frac{\sin 2x}{-4D}$$

$$= \frac{\sin 2x D}{-4D^2} = \frac{D(\sin 2x)}{-4(-2^2)}$$

$$= \frac{2\cos 2x}{16} = \frac{\cos 2x}{8}$$

61. (A)

By Bayes' theorem, suppose A_1, A_2, ... A_n are n mutually exclusive and exhaustive events such that $p(A_i)>0$ for i=1, 2, ... n.

Let B be any event with P(B)>0, then

$P(A_i)\ P(B/A_i)$

$P(A_i/B) =$

$$\frac{p(A_i)p(B/A_i)}{p(A_i)p(B/A_1)+p(A_2)p(B/A_2)+\dots+p(A_n)p(B/A_n)}$$

$$= \frac{p(A_i)p(B/A_i)}{\sum_{j=1}^{n} p(A_j)p(B/A_j)}$$

62. (C)

Since f(x) is a pdf of a continuous random variable x,

$$\int_{-\infty}^{\infty} \frac{A}{\pi}\frac{1}{x^2+16}dx = 1$$

$$\Rightarrow \frac{A}{\pi}\int_{-\infty}^{\infty}\frac{dx}{x^2+4^2} = 1$$

$$= \frac{A}{\pi}\cdot\frac{1}{4}\left[\tan^{-1}\left(\frac{x}{4}\right)\right]_{-\infty}^{\infty} = 1$$

$$= \frac{A}{4\pi}\left[\tan^{-1}(\infty)-\tan^{-1}(-\infty)\right] = 1$$

$$= \frac{A}{4\pi}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = 1$$

$$\frac{A}{4\pi}[\pi] = 1$$

$$\therefore A = 4$$

64. (D)

Mean = np = 5 ... (1)

Standard deviation $\sqrt{npq} = 2$

$\Rightarrow npq = 4$ ... (2)

$$(2)\div(1) \Rightarrow \frac{npq}{np} = \frac{4}{5}$$

$$q = \frac{4}{5}$$

$$\therefore p = 1-\frac{4}{5} = \frac{1}{5}$$

$$(1) \Rightarrow \quad n\times\frac{1}{5} = 5$$

$$\therefore n = 25$$

$$\therefore (n, p) = \left(25, \frac{1}{5}\right)$$

65. (B)

$$\text{Var }(4X+3) = 4^2\text{ Var}(X) = 16\text{ Var}(X)$$

66. (B)

In a poisson distribution

Mean = E(X) = λ

Variance = λ

Now Var(X) = $E(X^2)-E(X)^2$

$\Rightarrow \lambda = 30-l^2$

$\therefore \lambda^2+\lambda-30 = 0$

$\Rightarrow (\lambda+6)(\lambda-5) = 0$

$\Rightarrow \lambda = -6, 5$

Since $\lambda>0$ implies $\lambda = 5$

$\therefore$ Variance $= \lambda = 5$

67. (D)

In a normal distribution,

Mean = Median = Mode = μ

68. (A)

$E(3X^2-2X)$

$= 3E(X^2)-2E(X)$

$$= 3\int_0^2 x^2.\frac{x}{2}dx - 2\int_0^2 x.\frac{x}{2}dx$$

$$= \frac{3}{2}\int_0^2 x^3dx - \int_0^2 x^2dx$$

$$= \frac{3}{2}\left[\frac{x^4}{4}\right]_0^2 - \left(\frac{x^3}{3}\right)_0^2$$

$$= \frac{3}{2}\left(\frac{16}{4}-0\right)-\left(\frac{8}{3}-0\right)$$

$$= 6-\frac{8}{3} = \frac{18-8}{3} = \frac{10}{3}$$

69. (A)

For Chi–Square distribution

$\mu_r' = n(n+2)\dots(n+2r-2)$

$E(X) = \mu' = n$

$\therefore$ Mean $= E(X) = n$

$\mu_2' = n(n+2)$

Variance $= \mu_2'-(\mu')^2$

$= n(n+2)-n^2$

$= n^2+2n-n^2 = 2n$

71. (A)

By Lagrange's theorem, if G is a finite group and H is a subgroup of G, then O(H) is a divisor of O(G).

73. (D)

Let $O(G) = 11^2\ 13^2$

The number of 11–sylow subgroups are in the form 1+11k

Also this must divide $11^2\ 13^2$

Since 1+11k is prime to 11^2, it must divide 13^2

13^2 has no factors in the form 1+11k

Therefore 1+11k = 1

Therefore there is only one 11–sylow subgroup in G.

Since all 11–sylow subgroups are conjugate, implies

that 11–sylow subgroup is normal in G

By same arguments as above.
We can conclude that there is only one 11–sylow subgroup which is normal in G.

75. (A)
$r(a) = \{x \in R / ax = 0\}$
Let $x, y \in r(a)$
then $ax=0$; $ay=0$
Now $a(x+y) = ax+ay=0+0=0$
$\therefore x+y \in r(a)$ $\therefore$ r(a) is associative
Also r(a) is associative
Since $a(0) = 0 \Rightarrow 0$ is the identity element.
Let $x \in r(a) \Rightarrow ax=0$
Now $a(-x) = -(ax)$
$= -0 = 0$
$\therefore (-x) \in r(a)$
$\therefore$ r(a) is an abelian group
Now let $x \in r(a) \Rightarrow ax=0$
and $\propto \in R$
Then $a(x\propto)$
$= (ax)\propto = 0\propto = 0$
$\therefore x\propto \in r(a)$
Therefore r(a) is a right ideal of R.

77. (A)
If $(vT, vT) = (v, v)$ for all $v \in V$, then T is unitary.

82. (B)
Cauchy's Integral formula :
Let f(z) be an analytic function in a simply connected domain D bounded by a simply closed curve C and is continuous on C. Then

$$f(z) = \frac{1}{2\pi i}\int_c \frac{f(\varepsilon)\,d\varepsilon}{\varepsilon - z}$$

where $Z \in D$. Therefore Cauchy's integral formula gives the value of the function at an interior point.

83. (D)
$z = x+iy \Rightarrow \bar{z} = x - iy$
$dz = dx+idy$
$z = t^2+it \Rightarrow x+iy=t^2+it$
$\therefore x = t^2$; $y = t$
$dx = 2t$; $dy = dt$
Now $z=0$
$\Rightarrow t^2+it = 0$
$\therefore x=t_2=0$; $y=t=0$
$\Rightarrow t = 0$
$z = 4+2i$
$x = t^2 = 4$
$t = 2$

$$\therefore \int \bar{z}dz = \int_0^2 (t^2 - it)(2tdt + idt)$$

$$= \int_0^2 (t^2 - it)(2t + i)dt$$

$$= \int_0^2 (2t^3 + it^2 - 2it^2 + t)dt$$

$$= \int_0^2 (2t^3 + t - it^2)dt$$

$$= \left[\frac{2t^4}{4} + \frac{t^2}{2} - \frac{it^3}{3}\right]_0^2$$

$$= \frac{2(2)^4}{4} + \frac{(2)^2}{2} - \frac{i(2)^3}{3}$$

$$= 8 + 2 - \frac{8i}{3}$$

$$= 10 - \frac{8i}{3}$$

85. (C)

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots..$$

$$(z-2)\ \sin\frac{1}{z+1}$$

$$= (z-2)\left(\frac{1}{z+1} - \frac{1}{(z+1)^3 3!} + \frac{1}{(z+1)^5 5!} - \ldots.\right)$$

Principal part $\left(\text{Powers of } \frac{1}{z+1}\right)$ has infinite number of terms. Therefore $z=-1$ is an essential singularity.

86. (C)
$1 < |z+1| < 2$

$$\Rightarrow \frac{|z+1|}{2} < 1$$

Now $\dfrac{1}{(z+1)(z+3)} = \dfrac{1}{(z+1)(2+(z+1))}$

$$= \frac{1}{2(z+1)\left(1 + \frac{(z+1)}{2}\right)}$$

$$= \frac{1}{2(z+1)}\left[1 - \frac{(z+1)}{2} + \frac{(z+1)^2}{4} - \frac{(z+1)^3}{8} + \frac{(z+1)^4}{16} - \ldots\right]$$

$$= \frac{1}{2(z+1)} - \frac{1}{4} + \frac{(z+1)}{8} - \frac{(z+1)^2}{16} + \ldots$$

87. (A)
Formula :
If $z=a$ is a pole of order n then residue of f(z) at $z=a$ is

$$\frac{1}{(n-1)!}\lim_{z\to a}\frac{d^{n-1}}{dz^{n-1}}\left((z-a)^n f(z)\right)$$

Now $\dfrac{1}{(z^2+1)^3} = \dfrac{1}{(z+i)^3(z-i)^3}$
$\therefore z=i$ is a pole of order 3.
Residue

$$= \frac{1}{(3-1)!}\lim_{z\to i}\frac{d^2}{dz^2}(z-i)^3\frac{1}{(z+i)^3(z-i)^3}$$

$$= \frac{1}{2}\lim_{z\to i}\frac{d^2}{dz^2}\frac{1}{(z+i)^3}$$

$$= \frac{1}{2}\lim_{z\to i}\frac{d^2}{dz^2}(z+i)^{-3}$$

$$= \frac{1}{2}\lim_{z\to i}\frac{d}{dz}(-3)(z+i)^{-4}$$

$$= \frac{-3}{2}\lim_{z\to i}(-4)(z+i)^{-5}$$

$$= 6\lim_{z\to i}\frac{1}{(z+i)^5}$$

$$= \frac{6}{(i+1)^5} = \frac{6}{(2i)^5} = \frac{6}{32i}$$

$$= \frac{-3i}{16}$$

88. (D)
If A and B are independent events then $p(A \cap B) = p(A).p(B)$
Now $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
$= p(A) + p(B) - p(A).p(B)$

$\Rightarrow 0.8 = 0.5+p(B)-0.5\ p(B)$
$\Rightarrow 0.3 = p(B)(1-0.5) = 0.5\ p(B)$

$\Rightarrow p(B) = \dfrac{0.3}{0.5} = 0.6$

89. (C)
Required probability
$$= \frac{4}{52}\times\frac{3}{51} = \frac{1}{221}$$

90. (D)
$p(A\cup B) = p(A)+p(B)-p(A\cap B)$
$= 0.35+0.73-0.14$
$= 0.94$
$\therefore \overline{p(A\cup B)} = 1-p(A\cup B)$
$= 1-0.94$
$= 0.06$

91. (B)
By Schwart z inequality,
$|(x, y)|\le||x||\ ||y||$
d

93. (D)
$\{(1, 0, 0 \ldots. 0), (0, 1, 0, 0, \ldots 0) \ldots (0, 0, \ldots 0, 1)\}$ is a standard basis and it is an orthonormal subset of ℓ_2^n.

94. (D)
If $\{e_i\}$ is an orthonormal set in a Hilbert space.
Then $S = \{e_i : (x, e_i) \neq 0\}$ is countable

96. (B)
The spectrum $\sigma(x)$ is always non-empty.

98. (B)
$$a_n = \frac{n!}{n^n}$$
$$a_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$$
$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(n+1)^{n+1}}\times\frac{n^n}{n!}$$
$$= \frac{(n+1)n!}{(n+1)(n+1)^n}\times\frac{n^n}{n!}$$
$$= \frac{n^n}{(n+1)^n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$$
$$\Rightarrow \frac{1}{e} \text{ as } n\to\infty$$
$$\therefore \frac{1}{R} = \frac{1}{e}$$
$\therefore$ Radius of convergence = R = e
Region of convergence is $|z|<e$

99. (A)
Required cross ratio
$$= \frac{(z_1-z_2)(z_3-z_4)}{(z_2-z_3)(z_4-z_1)}$$
$$= \frac{(w_1-w_2)(w_3-w_4)}{(w_2-w_3)(w_4-w_1)} \quad \ldots(1)$$
If $w_2=0$; $w_3=1$; $w_4=\infty$
then (1) $\Rightarrow$
$$\frac{(z_1-z_2)(z_3-z_4)}{(z_2-z_3)(z_4-z_1)} = \frac{(w_1-0)(1-w_4)}{(0-1)(w_4-w_1)}$$
$$= \frac{w_1w_4\left(\frac{1}{w_4}-1\right)}{(-1)w_4\left(1-\frac{w_1}{w_4}\right)} \quad (\text{put } w_4 = \infty)$$
$= w_1$
Therefore the cross ratio (z_1, z_2, z_3, z_4) is w_1 (the image of z_1) under the linear transform which carries z_2, z_3, z_4 onto 0, 1, ∞.

100. (C)
Consider the transform
$$W=-i\left(\frac{z+1}{z-1}\right)$$
when $z=0$, $w=-i\left(\frac{0+1}{0-1}\right)=i$

when $z=-i$, $w=-i\left(\frac{-i+1}{-i-1}\right)$
$$= i\left(\frac{1-i}{1+i}\right)$$
$$= i\left(\frac{1-i}{1+i}\times\frac{1-i}{1+i}\right)$$
$= i(-i) = 1$

when $z=-1$, $w=-i\left(\frac{-1+1}{-1-1}\right)=0$

Therefore, under the bilinear transform $-i\left(\frac{z+1}{z-1}\right)$ maps $z=0, -i, -1$ into $w=i, 1, 0$.

101. (A)
The number of non-basic variables
$= mn-(m+n-1)$
$= 4\times5-(4+5-1)$
$= 20-(8)$
$= 12$

102. (B)
Mean time to failure for a health hazard model
$= \text{MTTF} = \dfrac{1}{\lambda}$

103. (A)
For series connection of 100 units each having reliability 0.98.
$= (0.98)\times(0.98)\times\ldots.$ 100 times
$= (0.98)^{100}$

104. (D)
Free float 3–4 = 18–12 = 6

106. (C)
Let M be a subspace of a normed linear space N.
Then $\{x+M/x\in N\}$ is a normed space in the quotient space if M is a closed subspace of N.
Also the quotient normed is defined as
$||x+M||=\inf\{||x+m||/m\in M\}$

107. (C)
Every continuous linear transform is bounded. Therefore if T is continuous linear transform then there exists areal number $k\ge0$
such that $||T(x)||\ k||x||\ \forall x\in N$

108. (B)
Let N be a normed linear space and x_0 is a non zero vector in N. Then there exists a continuous linear functional f_0 on X such that $f_0(x_0)=||x_0||$and $||f_0||=1$

110. (C)

Closed Graph Theorem :

Let X and Y be Banach spaces and let T be a linear transformation of X into Y. Then T is continuous if and only if its graph is closed in X×Y.

111. (C)

Duplin's Indicatrix

Definition :

The section of any surface by a plane parallel to and indefinitely, near the tangent plane at any point O on the surface is a conic section.

112. (D)

For asymptotic lines

$$\frac{\overrightarrow{dN}}{ds}.\frac{\overrightarrow{dr}}{ds}=0$$

$$\Rightarrow d\vec{N}.d\vec{r}=0$$

$$\Rightarrow d\vec{r}.d\vec{N}=0$$

113. (A)

A surface generated by one parameter family of planes is called a developable surface or simply a developable.

114. (D)

Canonical equations for geodesies are

$$u\equiv\frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right)-\frac{\partial T}{\partial u}=0$$

$$v\equiv\frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right)-\frac{\partial T}{\partial v}=0$$

115. (B)

$a_1x_1+a_2x_2 \geq b$

$-(a_1x_1+a_2x_2)\leq -b$

$\Rightarrow -a_1x_1-a_2x_2\leq -b$

117. (C)

Min z = CX

subject to

AX = b

Dual of this problems

Max $Z^* = b^TW$

subject w

$A^TW \leq C^T$ and W is unrestricted.

118. (D)

Pure strategies have a saddle point.

119. (B)

$$\begin{bmatrix}6 & -3\\ -3 & 0\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$$

Value of the game

$$=\frac{a_{11}a_{22}-a_{12}a_{21}}{a_{11}+a_{22}-(a_{12}+a_{21})}$$

$$=\frac{6\times 0-(-3)(-3)}{6+0-(-3-3)}$$

$$=\frac{-9}{12}=\frac{-3}{4}$$

121. (B)

By transformatiioni formula for the theta function,

$$\theta(x)=\sum_{n=-\infty}^{\infty}e^{-\pi n^2 x} \text{ then}$$

$$\theta(x)=\frac{1}{\sqrt{x}}\theta\left(\frac{1}{x}\right) \text{ for } x>0$$

122. (D)

By transform of f(x) is

$$F(f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)e^{ipt}dt$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a}f(t)e^{ipt}dt+\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}f(t)e^{ipt}dt$$

$$+\frac{1}{\sqrt{2\pi}}\int_{a}^{\infty}f(t)e^{ipt}dt$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}f(t)e^{ipt}dt$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}1.e^{ipt}dt$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{ipt}}{ip}\right]_{-a}^{a}$$

$$=\frac{1}{\sqrt{2\pi}}.\frac{1}{ip}\left[e^{ipa}-e^{ipa}\right]$$

$$=\frac{1}{\sqrt{2\pi}}.\frac{1}{ip}.2i\sin pa$$

$$=\sqrt{\frac{2}{\pi}}\frac{\sin pa}{p}$$

123. (B)

$$\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)g(x)dx=\sum_{n=1}^{\infty}(a_nA_n+b_nB_n)+\frac{1}{2}a_0A_0$$

(Parsvel's identity)

126. (B)

$R^2 = \lambda^2+\mu^2 = \rho^2+\sigma^2\rho'^2$

$= \rho^2+(\rho'\ \sigma)^2$

127. (D)

For all helices curvature bears a constant ratio with torsion.

129. (D)

$dS = |r_1du\times r_2dv|$

$= |r_1\times r_2|dudv$

$= Hdudv$

133. (A)

An orthonormal system of complex valued function on every interval of length 2π is given by

$$\phi_n(x)=\frac{e^{inx}}{\sqrt{2\pi}}=\frac{\cos nx+i\sin nx}{\sqrt{2\pi}}$$

n = 0, 1, 2, ...

134. (C)

$$a_n = \frac{1}{\pi}\int_0^{2\pi}f(t)\cos nt\,dt$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}f(t)\sin nt dt$$

135. (D)

Riez-Fischer Theorem :

Let be an orthonormal set on an interval I. Let $f\in L^2(I)$ and

$$f(x)\sim\sum_{n=0}^{\infty}C_n\phi_n(x)$$

then $\sum_{k=0}^{\infty}|C_k|^2=\|f\|^2$

136. (A)

Let $f\in L([0,\ 2\pi])$ and let f be periodic with period 2π.

Let $s(x)=\lim_{t\to 0^+}\frac{f(x+t)+f(x-t)}{2}$

whenever the limit exists. Then for each x, for which $S(x)$ is defined, the Fourier series generated by f is Cesaro summable and has (C, 1) sum $S(x)$.

137. (B)

The convolution of f and g is

$$h(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt$$

139. (A)

The convolution

$h(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt$ exists for every x in R and the function h so defined is bounded on R.

140. (A)

By Poisson summation formula

$$\sum_{n=-\infty}^{\infty} \frac{f(m+)+f(m-)}{2}$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)e^{-2\pi int}dt$$

Also each series being absolutely convergent.

141. (D)

If the field F has p^m elements then F is the spliting field of the polynomial $x^{p^m}-x$.

144. (A)

Bolzano-Weierstrass Theorem :

If a bounded set S in R^n contains infinitely many points then there is atleast one point in R^n which is an accumulation point of S.

145. (C)

Let $f:S\rightarrow T$ be a function from a metric space S to another metric space T.

If f is continuous on a compact subset of s then f is uniformly continuous.

146. (D)

F is continuous from S into T then for every closed set F in T, $f^{-1}(F)$ is closed in S.

147. (C)

Closed and bounded subset of R^n is compact.

Therefore G be an open covering of a compact set A, then there is a finite sub collection of G covering A.

148. (C)

Rolle's Theorem :

If a functiion f on [a, b] is continuous on [a b], differentiable on (a, b) and f(a)=f(b), then there exists atleast one interior point C such that $f'(c)=0$.

149. (B)

$$\int_0^4 [x]dx = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx + \int_3^4 [x]dx$$

$$= \int_0^1 o\,dx + \int_1^2 1dx + \int_2^3 2dx + \int_3^4 3dx$$

$$\int_1^2 dx + 2\int_2^3 dx + 3\int_3^4 dx$$

$= [x]_1^2 + 2[x]_2^3 + 3[x]_3^4$

$= (2-1)+2(3-2)+3(4-3)$

$= 1+2+3$

$= 6$

150. (C)

$L(p^* f) \geq L(p, f)$